MICRO-428: Metrology

Edoardo Charbon, Claudio Bruschini, Georg Fantner
Institute of Electrical and Micro Engineering (IEM), School of Engineering (STI)
Ecole polytechnique fédérale de Lausanne (EPFL), Neuchâtel, Switzerland

TAs: Samuele Bisi, Alexandre Domenech

EPFL

MICRO-428: Metrology

Week Ten: Electrical Metrology

Claudio Bruschini
TA: Samuele Bisi
Edoardo Charbon
Advanced Quantum Architectures Laboratory (AQUA)
EPFL at Microcity, Neuchâtel, Switzerland

EPFL

Reference Books

- B. Razavi, *Design of Analog CMOS Integrated Circuits, McGraw Hill., 2017*
- C. Kittel, *Elementary Statistical Physics*, John Wiley, 1958

- E. Charbon, "*Imaging Sensors – ET 4390 Course Slides*", Delft 2016
- S. Cova, "*Sensors Signals and Noise – Course Slides*", Politecnico di Milano 2016 (Noise1, 2, 3, HPF1)
  -> link: http://home.deib.polimi.it/cova/elet/lezioni/lezioni.htm
- I. Rech, "*Signal Recovery 2021-2022 – Course Slides*", Politecnico di Milano (HPF1 – SR13)
  -> link: https://rech.faculty.polimi.it/?page_id=235

In the case of this lecture, there isn't a single source of most of the material. Instead, we used a collection of sources, in particular the Razavi book, as well as the three sources below, including the online ones from the Politecnico di Milano.

In general, see also the reference box at the bottom of the slides for notes on the exact chapters, etc.

Week 9 Summary

S

9.0 Random Variables/2: Mean, 2nd Order Moment and Variance of all previously introduced distributions (8.2), plus Gamma: $Y \sim \mathrm{Gamma}(a, \lambda)$, and some of their corresponding salient properties

9.1 Random (or stochastic) Process RP as a collection of an infinite number of Random Variables. Concept of Ensemble of signals (= set of all possible sample functions).

PDF $f_X(x,t)$ ↔ CDF $F_X(x,t)$, expected value (mean) $E\{X(t)\}=m_X(t) = \overline{X(t)}$, variance $Var\{X(t)\}$

$$= E\left\{\left(X(t) - m_X(t)\right)^2\right\} = \sigma^2$$

$$\rightarrow C_{XX}(t_1, t_2), K_{XX}(t_1, t_2)\text{: auto-covariance/correlation}$$

$$\rightarrow C_{XY}(t_1, t_2), K_{XY}(t_1, t_2)\text{: cross-covariance/correlation}$$

The lecture starts with a small recap of the main elements of the previous week.

Week 9 Summary

S

9.1 Stationary RP: statistical properties do not change in time. Weaker form: Wide-Sense Stationary, constant mean + $K_{XX}(t, t+\tau) = K_{XX}(\tau)$

Ergodic random processes: statistical properties can be deduced from a single, sufficiently long, random sample: Ensemble average ↔ Time average

$$\overline{X(t)} = E\{X(t)\} = \langle X(t)\rangle, K_{XX}(\tau) = E\{X(t)\cdot X(t+\tau)\} = \mathcal{K}_{XX}(\tau)$$

9.2 Law of Large Numbers: as n grows, the sample mean $\overline{X_n}$ converges to the true mean μ

$$E\{\overline{X_n}\} = \mu, \mathrm{Var}\{\overline{X_n}\} = \frac{\sigma^2}{n}, \text{assuming i.i.d. } X_1, X_2 X_3 \ldots \text{RVs}$$

Central Limit Theorem: Sum of a large number of i.i.d. random variables has an approximately Gaussian (normal) distribution

9.3 Elements of Estimation Theory: how do we use collected data to estimate unknown parameters of a distribution?

9.4 Accuracy (-> mean), Precision (-> spread), Resolution

Outline

10.1 Charges, Currents, and Voltages

10.2 Noise Background

10.3 Noise Sources

11.1 Noise Reduction, Averaging Techniques

11.2 Electric Signals, Analog-to-Digital Conversion

11.3 Timing – Time-to-Digital Conversion

12.1 Electrical Metrology Tools

The Outline covers both this lecture as well as the next ones.

10.1.1 Charge

S

- Electric charge is the physical property of matter that causes it to experience a force when placed in an electromagnetic field.
- Charge is measured in coulomb [unit: C] named after French physicist Charles-Augustin de Coulomb.
- The symbol Q often denotes charge.
- Proton and electron have equal and opposite elementary (indivisible) charge = 1.602 176 634 x 10^{-19} C (exact by definition of the coulomb, 20/05/2019)
  -> Experiments to determine the existence of fractional charges
  - Quarks: charge quantized into multiples of 1/3e (but cannot be seen as isolated particles)
  - Quasiparticles can also have fractional charges

Charles-Augustin de Coulomb

EN Wikipedia *Electric_charge*

The following set of 5 slides provides a recap of the basic (microscopic) definitions of the concepts of Charge, Current and Voltage.

10.1.1 Charge

S

- Conservation of Charge: Electric charge is a conserved property; the net charge of an isolated system, the amount of positive charge minus the amount of negative charge, cannot change.

-> Charge-current continuity equation: ρ = charge density, **J** = current density, I = net current, V = volume of integration, S = closed surface ∂V, Q = charge contained within V

$$-\frac{d}{dt}\int_V \rho\, dV = \oiint_{\partial V} \mathbf{J}\cdot \mathrm{d}\mathbf{S} = \int J\,\mathrm{d}S\cos\theta = I$$

$$I = -\frac{dQ}{dt}, Q = \int_{t_1}^{t_2} I\,dt$$

- How to measure charge:
  - Millikan's oil drop experiment (1909)
  - Shot noise (analyse the noise of a current), Walter H. Schottky

Charles-Augustin de Coulomb

- Charge transported by a constant current of 1 A in 1 s:
  $1\,\mathrm{C} = 1\,\mathrm{A} \times 1\,\mathrm{s}$
- Amount of excess charge on a capacitor of 1 F charged to 1 V
  $1\,\mathrm{C} = 1\,\mathrm{F} \times 1\,\mathrm{V}$

EN Wikipedia *Electric_charge*

10.1.2 Current

S

- An electric current is a flow of charge.

- Current is the rate at which charge is flowing in a circuit. It is the amount of charges that pass through any point of the circuit per unit time.

$$\text{Steady: } I = \frac{Q}{t}, \text{ general: } I = {}^{dQ}/_{dt}$$

- The conventional symbol for current is *I*, which originates from the French phrase *intensité du courant* (current intensity)

- The ampere is the base unit of electric current in the International System of Units (SI). It is named after André-Marie Ampère (1775–1836), French mathematician and physicist, considered the father of electrodynamics.

André-Marie Ampère

EN Wikipedia *Electric_current*

10.1.2 Current

S

Ohm's Law (linear materials, low frequency): $I = {}^{V}/_{R}$

Ohmic (Joule's) heating: $P = IV = I^2R = {}^{V^2}/_{R}$

- Microscopically, a current can be carried by a flow of electrons, of (electron) holes acting as positive carriers, of ions or other charged particles, etc. E.g. in a semiconductor: drift + diffusion

$$J = \sigma E + Dq\nabla n$$

- Higher frequencies: *skin effect* (higher current density towards the surface)

André-Marie Ampère

EN Wikipedia *Electric_current*

10.1.3 Voltage

S

- Voltage, electric potential difference, electric pressure or electric tension is the difference in electric potential between two points.

- The difference in electric potential between two points (i.e., voltage) in a static electric field is defined as the work needed per unit of charge to move a test charge between the two points.

$$\Delta V_{AB} = V(x_B) - V(x_A) = -\int_{r_0}^{x_B} \vec{E} \cdot d\vec{l} - \left(\int_{r_0}^{x_A} \vec{E} \cdot d\vec{l}\right) =$$
$$= -\int_{x_A}^{x_B} \vec{E} \cdot d\vec{l}$$

- In the International System of Units, the derived unit for voltage is named volt (*V*). The volt is named in honour of the Italian physicist Alessandro Volta (1745–1827), who invented the voltaic pile, possibly the first chemical battery.

Alessandro Volta amazes Napoleon with his battery.

Voltage across an inductor:

$$\Delta V = -L\frac{dI}{dt}$$

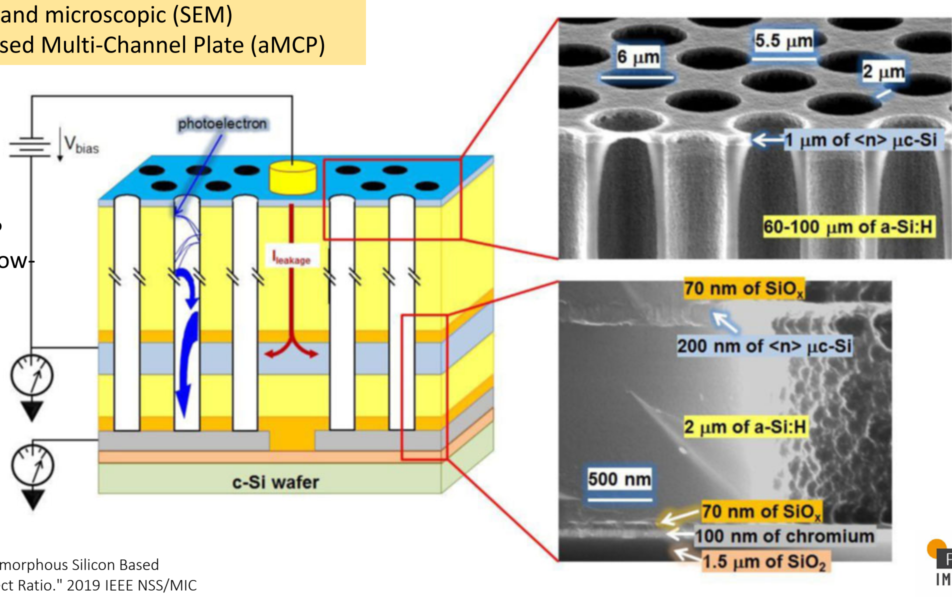
EN Wikipedia *Voltage*

Back to the Basics – Example Application

Rendering and microscopic (SEM) view of an a-Si:H based Multi-Channel Plate (aMCP)

Problem: estimate aMCP capacitance to optimize follow-up low noise amplifier

Q



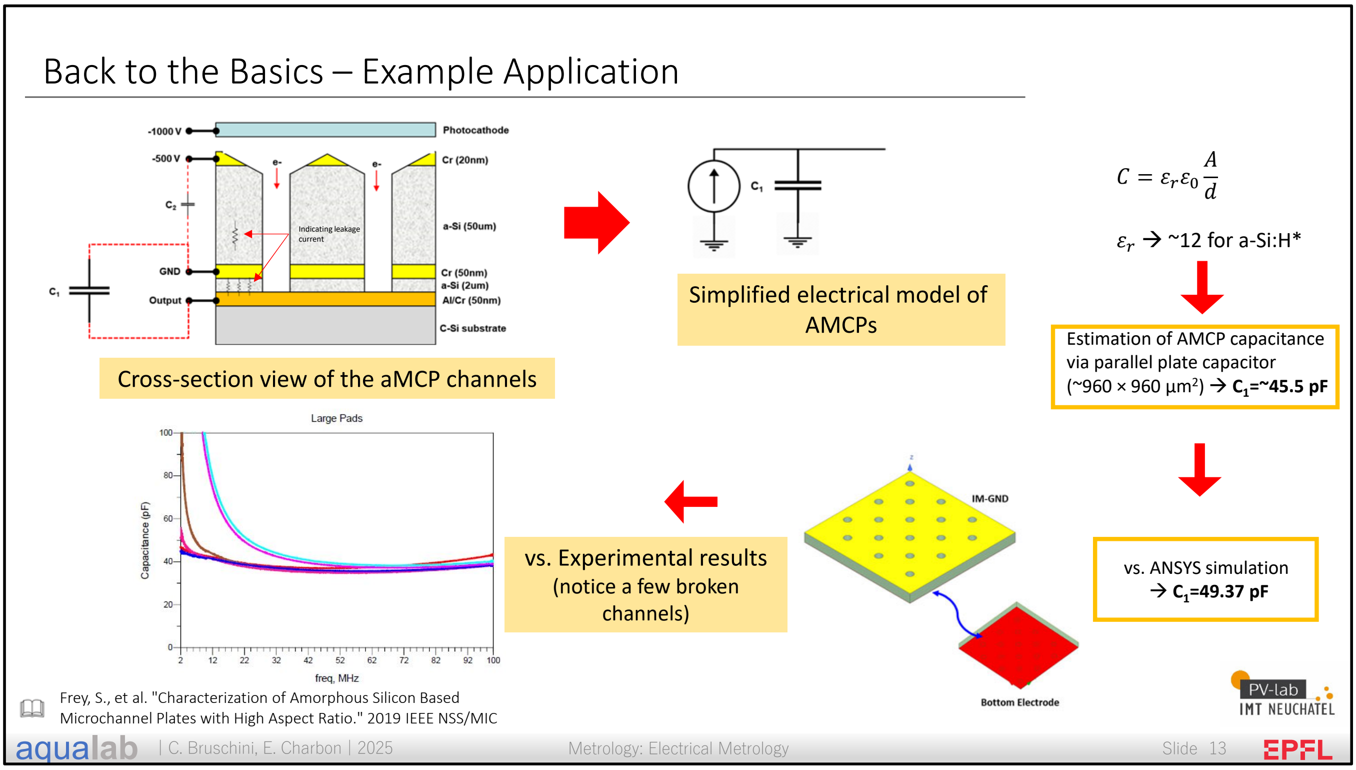
Frey, S., et al. "Characterization of Amorphous Silicon Based Microchannel Plates with High Aspect Ratio." 2019 IEEE NSS/MIC

PV-lab IMT NEUCHATEL

The following example is aimed at highlighting the power of "simple" estimates starting from first principles.

In this case, we need to estimate the capacitance of a multi-channel plate (MCP) electron multiplier, based on amorphous silicon (a-Si) and developed by colleagues at the EPFL PVLAB. The basic principle is the electron multiplication inside very small channels, due to the kinetic energy of the electrons hitting the channel walls, extracting secondary electrons. These are in turn accelerated, hit the walls, extract other secondary electrons and so on. The "first" electron is actually a photoelectron, extracted by a photon hitting a photocathode (not shown). The whole device operates under vacuum.

Hint: before going to the next slide, try to estimate the capacitance with a "back-of the-envelope" calculation!



Note how the simple estimate shown in the top row by means of an analogy with a parallel plate capacitor is already pretty close to the numerical result obtained through a finite element simulator (ANSYS, bottom right), and to the experimental results (bottom left), which obviously involve the full fabrication of a prototype!

Such an approach can be conceptually extended to a number of situations, where first estimates are pretty useful.

Outline

10.2 Sample Statistics – Wide-sense Stationary Noise (WSS)

- Recap: Noise is generally modeled as a random process *n*(*t*), i.e. a collection of random variables, one for each time instant *t* in interval]-∞,+∞[
- Each Random Variable (RV) has a probability density function *p*(*n*,*t*)
- In a stationary noise, *p*(*n*,*t*) is invariant in time
- Weaker form: in wide-sense stationary noise (WSS)...

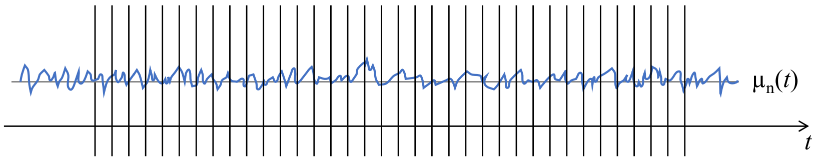
See also Section 9.1.1 & 9.1.2

(same message but seen from a noise perspective and adapted notation)

Mean : $E\{n(t)\} = \mu_n(t) = \mu_n(t+\tau)$ for all τ

Autocorrelation: $E\{n(t_1) \cdot n(t_2)\} = K_{nn}(t_1, t_2) = K_{nn}(t_1+\tau, t_2+\tau)$ for all τ

(the autocorrelation function only depends on the time difference, but not on the absolute position in time)



E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

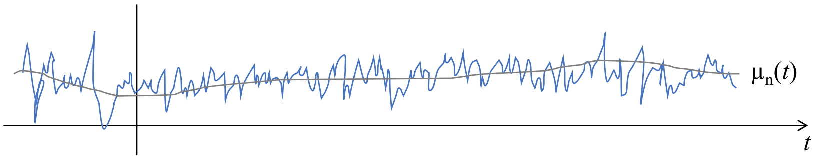
Random Process: we recall here the **same message as in Sections 9.1.1 & 9.1.2 but seen from a noise perspective and using an adapted, slightly different notation** (e.g. $K_{nn}(t_1+\tau, t_2+\tau)$ for the autocorrelation function, etc.).

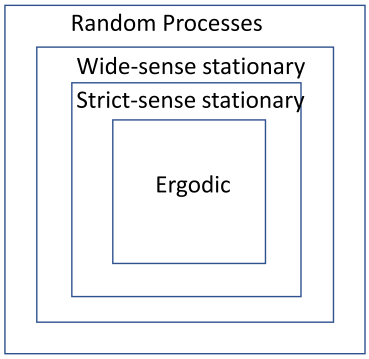
- $\mu_n(t)$ is the mean of the random variable (RV) *n*(*t*)
- $K_{nn}(t_1, t_2)$ is the autocorrelation of RVs $n(t_1)$ and $n(t_2)$

NB: Concerning the sample statistics: a collection of independent and identically distributed (i.i.d.) RVs implies a WSS process. Conversely, a WSS process does not necessarily imply a collection of i.i.d. RVs!

10.2 Time vs. Space

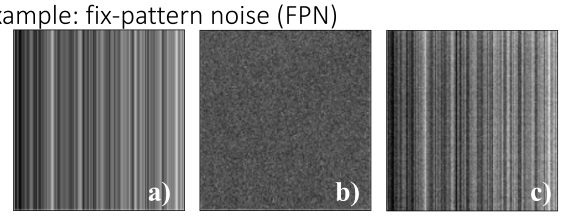
- In a non-stationary noise, $p(n,t)$ is variant in time





- A random process may be time-invariant but variant in space (x and/or y)

Example: fix-pattern noise (FPN)



Pictures from J Goy PhD (INP Grenoble, 2002)

a) FPN in x

b) FPN in y

c) FPN in x and y

📖 E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

FPN example: actually, the SNR of a) and b) is the same, but a) is much more disturbing to the human brain!

10.2 Noise Source Characterization

Recap: k-Moments of RV n at time t (but omitting the dependence)

$$m_k = E\{n^k\} = \int_{-\infty}^{\infty} n^k p(n)\, dn$$

Example:

$$m_0 = 1$$
$$m_1 = E\{n\} = \int_{-\infty}^{\infty} n\, p(n)\, dn = \mu_n$$
$$m_2 = E\{n^2\} = \int_{-\infty}^{\infty} n^2 p(n)\, dn$$

$$\sigma^2 = E\{n^2\} - \mu_n^2$$

Note: at each point in time t the RV n(t) will have exactly the same statistical properties if it is i.i.d.

See also Section 8.3

i.i.d. = independent and identically distributed Random Variables, have the same PDF and are all mutually independent

-> a collection of i.i.d. RVs implies a WSS process, but not vice versa (sufficient but not necessary condition).

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

Noise as a Random Process and how to characterise a noise source by its moments: this is again similar to what seen in the previous week, but omitting the dependence on time t (which is now implicit).

NB: we are implicitly using LOTUS (Week 8 8.3.3) in the calculation of $m_k = E\{n^k\}$ and the following ones.

____

On **white noise** [statistical view]:

- independent and identically distributed random variables (i.i.d RVs) are the simplest representation of white noise. In particular, if each sample has a normal distribution with zero mean, the signal is said to be additive white Gaussian noise. (https://en.wikipedia.org/wiki/White_noise)

- a "sequence of serially uncorrelated random variables with zero mean and finite variance" is actually sufficient (they don't necessarily need to be independent).

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10.2 Noise Power

Compute the autocorrelation of n at time t_1:

$$K_{nn}(t_1, t_1+\tau) = E\{n(t_1)\cdot n(t_1+\tau)\}$$

$$\text{call } n(t_1) = n_1 \text{ and } n(t_1+\tau) = n_2$$

See also Section 9.1.1 & 9.1.2

$$\rightarrow K_{nn}(t_1, t_1+\tau) = \int_{-\infty}^{\infty} n_1\, p(n_1)\, n_2\, p(n_2)\, dn_1\, dn_2$$

If $n(t)$ is a WSS Process, then:

$$K_{nn}(t_1, t_1+\tau) = K_{nn}(\tau)$$

i.e. the autocorrelation is independent of t_1. Follows that:

$$K_{nn}(0) = E\{n^2(t)\} = \overline{n^2(t)} \quad (= \sigma_n^2 \text{ assuming 0 mean: } \mu_n = 0)$$

This is the noise power!

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

Note that for a WSS noise, the **noise power** $E\{n^2(t)\} = \overline{n^2(t)}$, which is a quantity of fundamental importance in the characterisation of noise sources, does not change over time – it is always equal to the value of the autocorrelation function at zero delay, $K_{nn}(t_1, t_1) = K_{nn}(0)$. This illustrates the importance of the autocorrelation function.

NB:

- $K_{nn}(t_1, t_1+\tau)$ is actually a double integral, here we simplify the notation.

- $E\{XY\}$ – i.e. the product of two RVs – is calculated in Week 8 (8.4.4).

10.2 Note on the Power of a Signal

- The average power P of a signal $x(t)$ is defined as:

$$P = \lim_{T\to\infty} \frac{1}{T} \int_0^T |x(t)|^2 dt$$

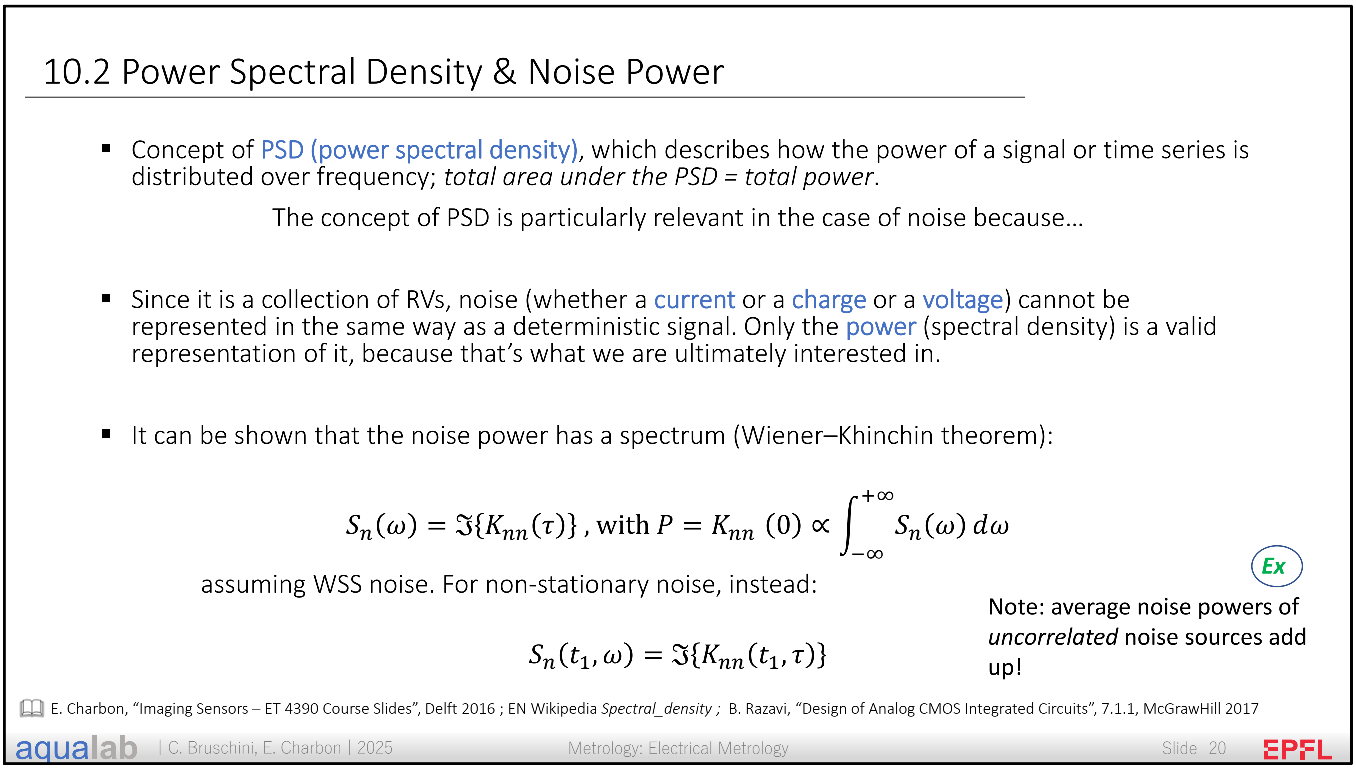
NB: the power can be the actual physical power, or more in general, the squared value of the signal

(think also in terms of a hypothetical voltage source which followed $x(t)$ applied to the terminals of a 1 Ohm resistor -> instantaneous power dissipated in that resistor would be $|x(t)|^2$ Watt).

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016 ; EN Wikipedia *Spectral_density*

As a sidenote, why do we speak of "**power of a signal**"?

In this "parenthesis" we illustrate the general concept of power of a signal (actually for an "ordinary" signal in the time domain) and introduce in the next slide the concept of Power Spectral Density (PSD).

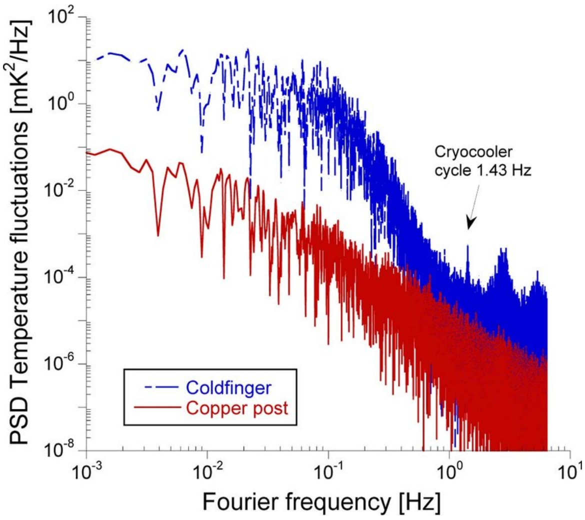


Once the concept of power of a signal has been introduced, it's straightforward to define the **Power Spectral Density (PSD)** and explain why it is of particular relevance in the case of noise (second bullet):

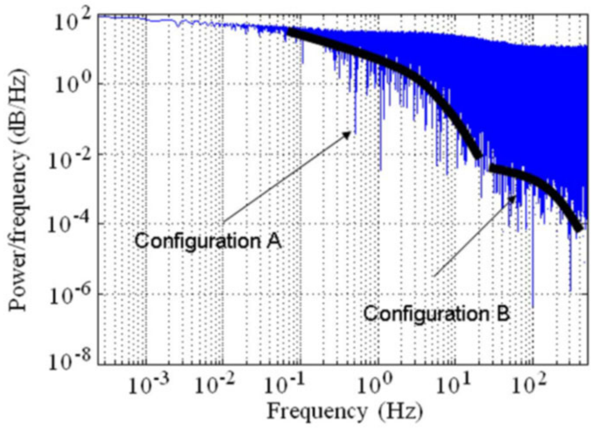
[Razavi] "What we meant is that the most appropriate representation of noise is usually by referring to its power and PSD (Power Spectral Density), because that's what we are ultimately interested in due to the underlying nature of noise as a random process (= collection of RVs → statistical description).
This as opposed to a deterministic signal, where its instantaneous value in the time domain can be predicted from the observed waveform, whereas for noise in general it can not (but the power usually can, at least for stationary processes)."

The Wiener–Khinchin theorem (third bullet - https://en.wikipedia.org/wiki/Wiener%E2%80%93Khinchin_theorem) does then link the PSD with the autocorrelation function, and thus the noise power (for a WSS process). We can thus work in the frequency domain or the time domain.

10.2 Power Spectral Density Examples



RTS PSD example (slide 58)

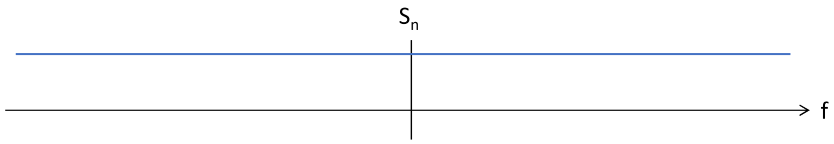


J. G. Hartnett & N. R. Nand, IEEE TMTT(58), 2010. M. A. Karami et al., *Random Telegraph Signal in Single-Photon Avalanche Diodes*, International Image Sensor Workshop, Bergen, 2009

These are examples of actual PSD data, here in the low frequency domain. The left one is of mechanical origin, the right one electrical and linked to slide 58 (irradiated SPAD photodetectors). Note the clear contributions from two distinct device configurations, which are shown in slide 58 in the time domain.

10.2 Noise Power Spectrum – White Noise

- Common type of noise PSD (power spectral density) is white noise.
- Displays same value at all frequencies. *Q*
- White noise does not exist strictly speaking since total power carried by noise cannot be infinite.
- Noise spectrum that is flat *in the band of interest* is usually called white.



B. Razavi, "Design of Analog CMOS Integrated Circuits", McGrawHill 2017

Let's move to a common example, i.e. white noise, characterised by a flat behaviour (see graph).

Q: does this kind of flat spectrum make (physical) sense?

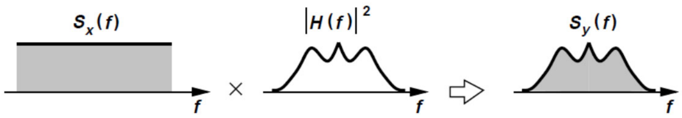
10.2 Noise Power Spectrum – Theorem

The PSD is a powerful tool in analyzing the effect of noise in circuits, especially in conjunction with the following Theorem:

- If a signal with spectrum $S_x(f)$, i.e. input PSD, is applied to a linear time-invariant (LTI) system with transfer function $H(s)$, then the output spectrum $S_Y(f)$, i.e. output PSD, is given by:

$$S_Y(f) = S_x(f)|H(f)|^2 \qquad \text{where } H(f) = H(s = j2\pi f)$$

- In other words, the spectrum of the signal is shaped by the transfer function of the system.



B. Razavi, "Design of Analog CMOS Integrated Circuits", 7.1.1, McGrawHill 2017

How is noise, and in particular its spectrum (**PSD**), affected by a system?

The special case of a **linear time-invariant system (LTI)** is of particular interest. In general:

$y(t) = x(t) * h(t) \Leftrightarrow Y(f) = H(f) \cdot X(f)$,
[* = convolution; (*left*) time domain & impulse response function $h(t)$ vs. (*right*) frequency domain & transfer function $H(f)$]

But this does also apply to noise/a random process, i.e. if *x* and *y* are random processes. This is a priori not obvious, and leads to the following link between input and output PSD: $S_Y(f) = S_x(f)|H(f)|^2$. *x* might be signal plus noise, or just noise.

Source: H. Bilgekul, Slides for the course "EE-461 Communication System II", EMU

Razavi and others employ Laplace transforms (https://en.wikipedia.org/wiki/Linear_time-invariant_system).

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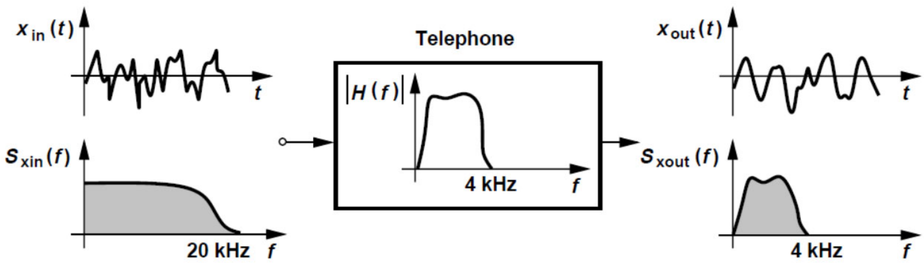
10.2 Noise Power Spectrum – Theorem Example

- Regular [*old…*] telephones have a bandwidth of approximately 4 kHz and suppress higher frequency components in caller's voice.



- Due to limited bandwidth, $x_{out}(t)$ exhibits slower changes than $x_{in}(t)$, in which case it can be difficult to recognize the caller's voice.

B. Razavi, "Design of Analog CMOS Integrated Circuits", McGrawHill 2017

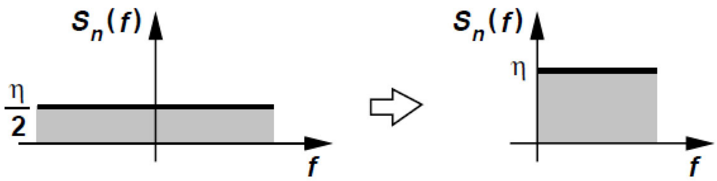
This example is a bit dated, but still illustrates well the effect of the transfer function on the signal itself (top row) *as well as* on the input PSD / bottom row.

10.2 Noise Power Spectrum – Bilateral vs. Unilateral Spectra

- $S_n(f)$ is an even function of f for real $n(t)$.
- PSD (power spectral density) can be specified as one-sided (only positive frequencies; typ. engineering) or two-sided functions (positive and negative frequencies; 2x smaller; typ. physics):

negative frequency part of the spectrum is folded around the vertical axis and added to the positive frequency part.

- Example: two sided white spectrum can be folded around the vertical axis to give a one sided white spectrum:



B. Razavi, "Design of Analog CMOS Integrated Circuits", McGrawHill 2017

Note the need to specify whether the PSD has been defined as two-sided or one-sided, and the different conventions depending on the community.

Noise Analysis Procedure

- Identify the sources of noise, determine each spectrum
- Find the transfer function from each noise source to the output (as if the source were a deterministic signal)
- Utilize the theorem $S_Y(f) = S_x(f)|H(f)|^2$ to calculate the output noise spectrum contributed by each noise source. (The input signal is set to zero.)
- Add all of the output spectra, paying attention to correlated and uncorrelated sources

→ Output noise spectrum, to be integrated from -∞ to +∞ to obtain the total output noise

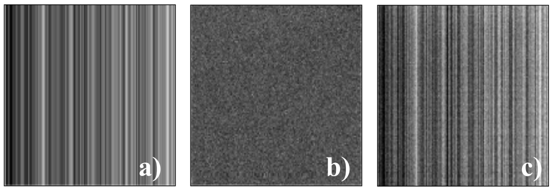
→ We need the noise representation of various sources…

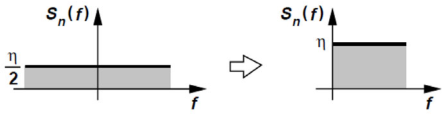
B. Razavi, "Design of Analog CMOS Integrated Circuits", 7.1.5, McGrawHill 2017

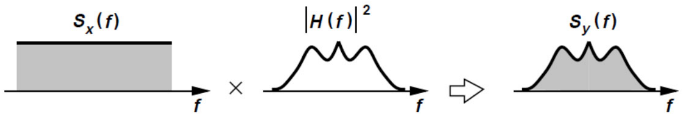
The description of the general noise analysis procedure provides us with a natural link to the **need to know the noise representation of various sources**, which will be done in the next sections.

Take-home Messages/W4-1

- *Noise modelled as a Random Process:*
  - Time vs. Space
  - Wide-sense Stationary Noise (WSS)
  - Noise Source Characterization
    - *n*-Moments
    - Noise Power and Noise Power Spectrum -> Power Spectral Density (PSD)
    - White Noise, Bilateral vs. Unilateral
    - LTI (linear time-invariant) systems & noise analysis procedure







First recap section: we summarise here the main definitions, results and examples discussed so far. They should be clear and understood.

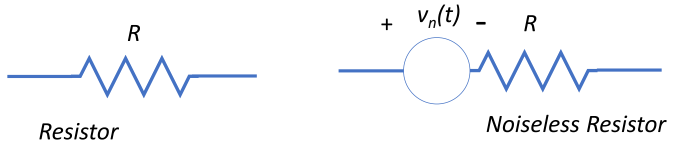
Outline

We now tackle the description and specificities of several important noise sources, shown in the next slide.

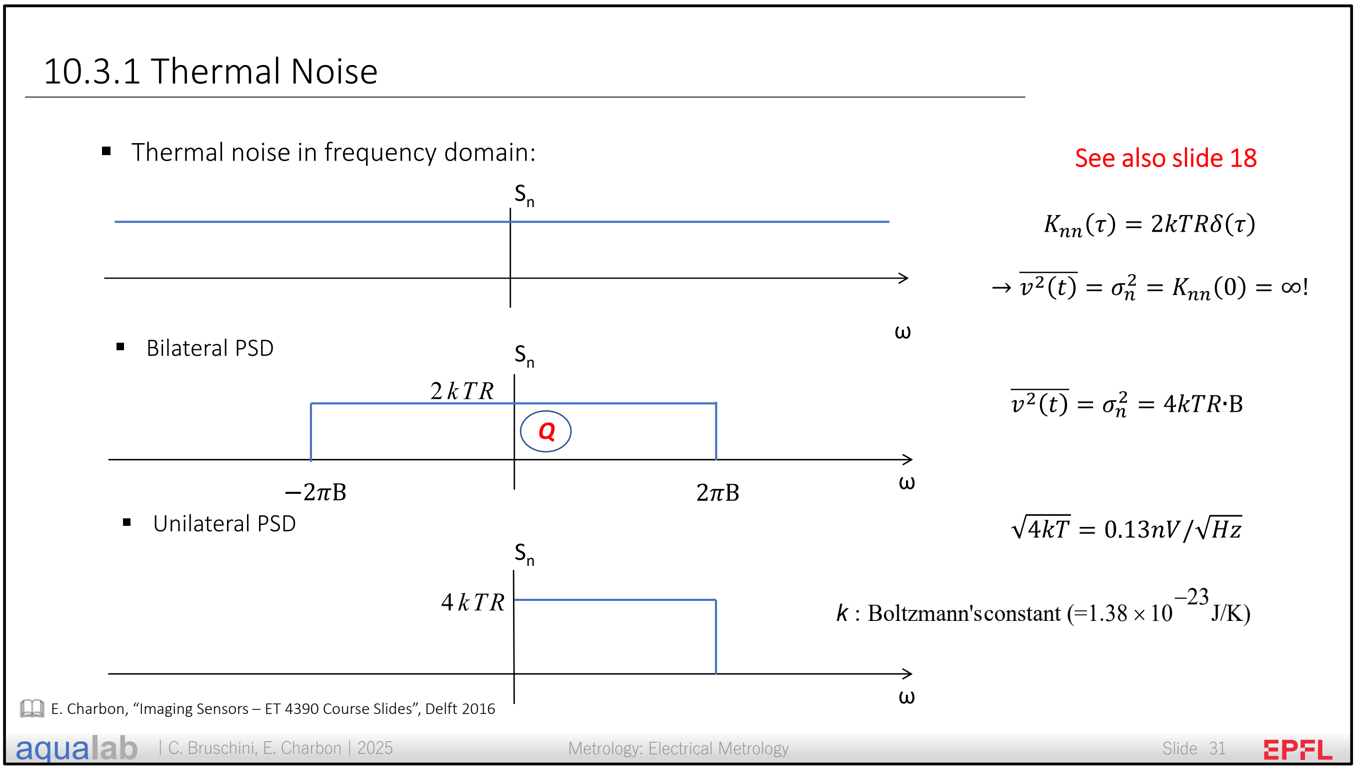
10.3 Noise Sources

10.3.1 Thermal Noise

- Thermal noise is observed in any system having thermal losses and is caused by thermal agitation of charge carriers.
- An example of thermal noise can be in resistors. Random motion of electrons in a resistor induces fluctuations in the voltage measured across it even though the average current is zero.
- Thermal noise is also called as Johnson-Nyquist Noise.
- Thermal noise on a resistor can be modeled by a series voltage source $v_n(t)$. *Ex*

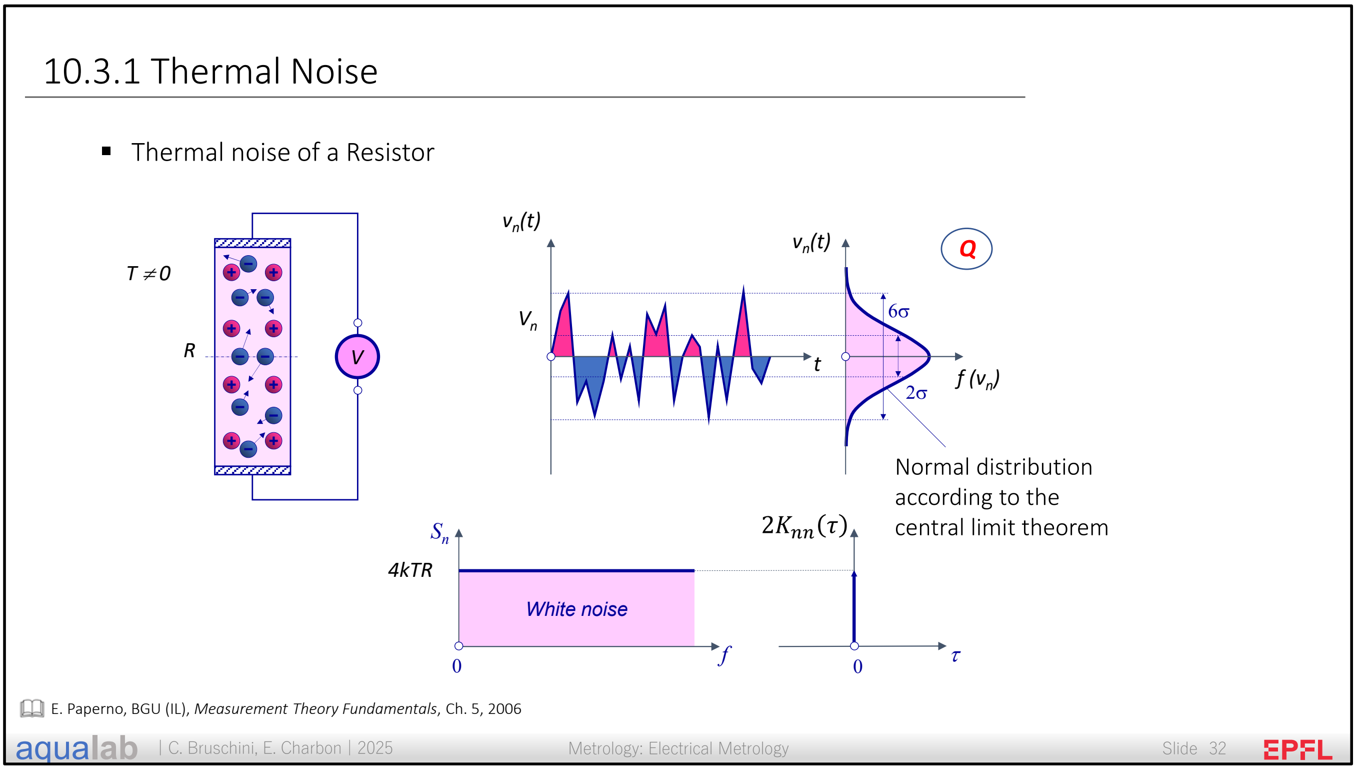


C. Kittel, *Elementary Statistical Physics*, John Wiley, 1958 ; E. Paperno, BGU (IL), *Measurement Theory Fundamentals*, Ch. 5, 2006 -> K. B. Klaassen, Electronic measurement and instrumentation, Cambridge University Press, 1996.



Q: look at the 2nd image – Bilateral PSD – and think of an example of a signal which has a strong spectral component close to the origin. Then suppose that we are filtering the total signal plus noise using a filter of bandwidth *B* (see also the next lecture) – is the choice of *B* a good one?

→ Likely not, because we would be integrating a lot of noise.



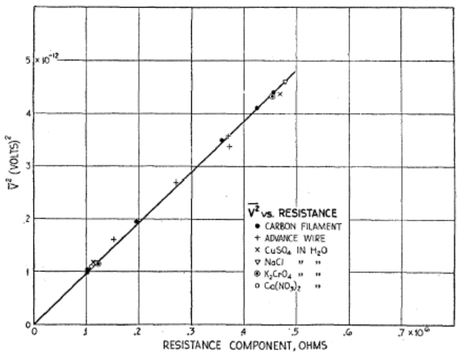
How does **thermal noise** look like in the time domain?

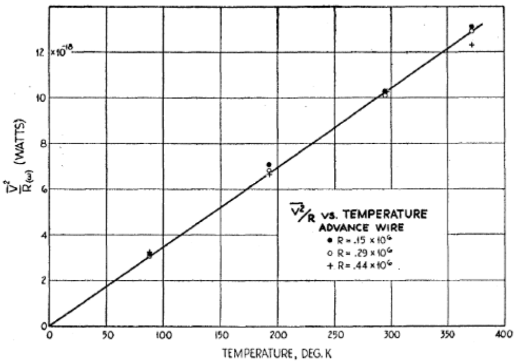
$v_n(t)$ behaves in time as shown in the central plot. The (rotated) histogram of the $f(v_n)$ values on the right indicates the probability distribution of the voltage values across the resistor, i.e. the PDF of v_n(t).

The *RV* is here the (noise) voltage, with standard deviation $\sigma_n = \sqrt{4kTR \cdot \mathrm{B}}$. The PDF is Gaussian – it should not to be confused with the thermal noise spectrum, or PSD, shown on the previous slide, which is flat!

10.3.1 Thermal Noise (Johnson Experiment)

Thermal Noise of a *Resistor*





The thermal noise PSD is:

- Proportional to the resistance
- Proportional to the temperature

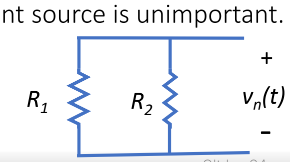
J.B. Johnson, "Thermal Agitation of Electricity in Conductors, Physical Review, vol. 32, pp. 97-109, 1928

Plots from the original Johnson 1928 paper – note the experimental errors!

10.3.1 Thermal Noise - Summary

- Only resistance creates thermal noise.
- Ideal capacitors and inductors do not generate any thermal noise.
- However, they do accumulate noise generated by other sources -> see kT/C noise....
- A noise source with a power spectrum that is flat and has a infinite bandwidth is called white noise.
- In practice, the bandwidth is never infinite but always limited.
- Since noise is a random quantity, the polarity used for the voltage or current source is unimportant.

Q How to reduce it?



Ex

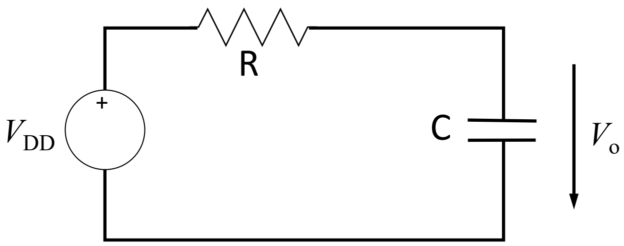
Q: if thermal noise is the dominant noise source, then why not simply cool?

→ There might be other noise sources which then become dominant, and in general it's not as simple as taking a piece of electronics and cooling it down. In general, cooling requires power as well.

In the specific case of cryo-CMOS, think of the changes taking place in the device response parameters (e.g. transistor I-V curves, resistor and capacitance values).

10.3.2 kTC Noise – Thermal Noise on Capacitors

- An ideal switch goes from closed (zero resistance) to open (infinite resistance) in zero time and hence would not create kTC noise.
- When a real switch is opened, it must have a resistance (even for a short amount of time):



RC circuit!

$$\overline{V_0^2(\omega)} = \left|\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R}\right|^2 \overline{v^2(\omega)} \xrightarrow{\text{Parseval}} \overline{V_0^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{1}{1+j\omega CR}\right|^2 \cdot \frac{4kTR}{2}\,d\omega$$

Parseval's theorem:

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

kTC noise (also written kT/C, and called *reset noise*): a real switch has a contact resistance, but from a microelectronics perspective, think in terms of a transistor used as a switch. This type of noise is for example quite important in imaging sensors.

NB: V_0 is the voltage which will appear across the capacitance. The left voltage generator + noiseless resistor *R* is the equivalent representation of a real resistor (= switch here).

Mathematically:

- We use Parseval's theorem to switch to the frequency domain and use the frequency domain circuit response on the left.
- The $4kTR/2$ factor comes from the white noise bilateral PSD of a thermal noise source!

10.3.2 kTC Noise

Finally:

$$\overline{V_0^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{1}{1+j\omega CR}\right|^2 \cdot \frac{4kTR}{2}\, d\omega$$

$$\overline{V_0^2} = \frac{kTR}{\pi}\int_{-\infty}^{\infty}\frac{1}{1+\omega^2C^2R^2}\, d\omega$$

$$\overline{V_0^2} = \frac{kTR}{CR\pi}\int_{-\infty}^{\infty}\frac{1}{1+x^2}\, dx = \frac{kTR}{CR\pi}[\arctan(x)]_{-\infty}^{\infty} = \frac{kTR}{CR\pi}\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = \frac{kT}{C}$$ Q

So, no matter how short time the resistance is in place (nor its value), this noise will appear! It is entirely due to thermal noise in the resistor. Ex

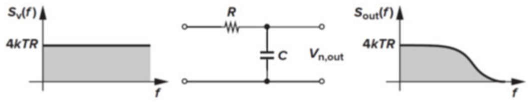


Figure 7.16 Noise spectrum shaping by a low-pass filter.

RC circuit -> low-pass noise shaping!

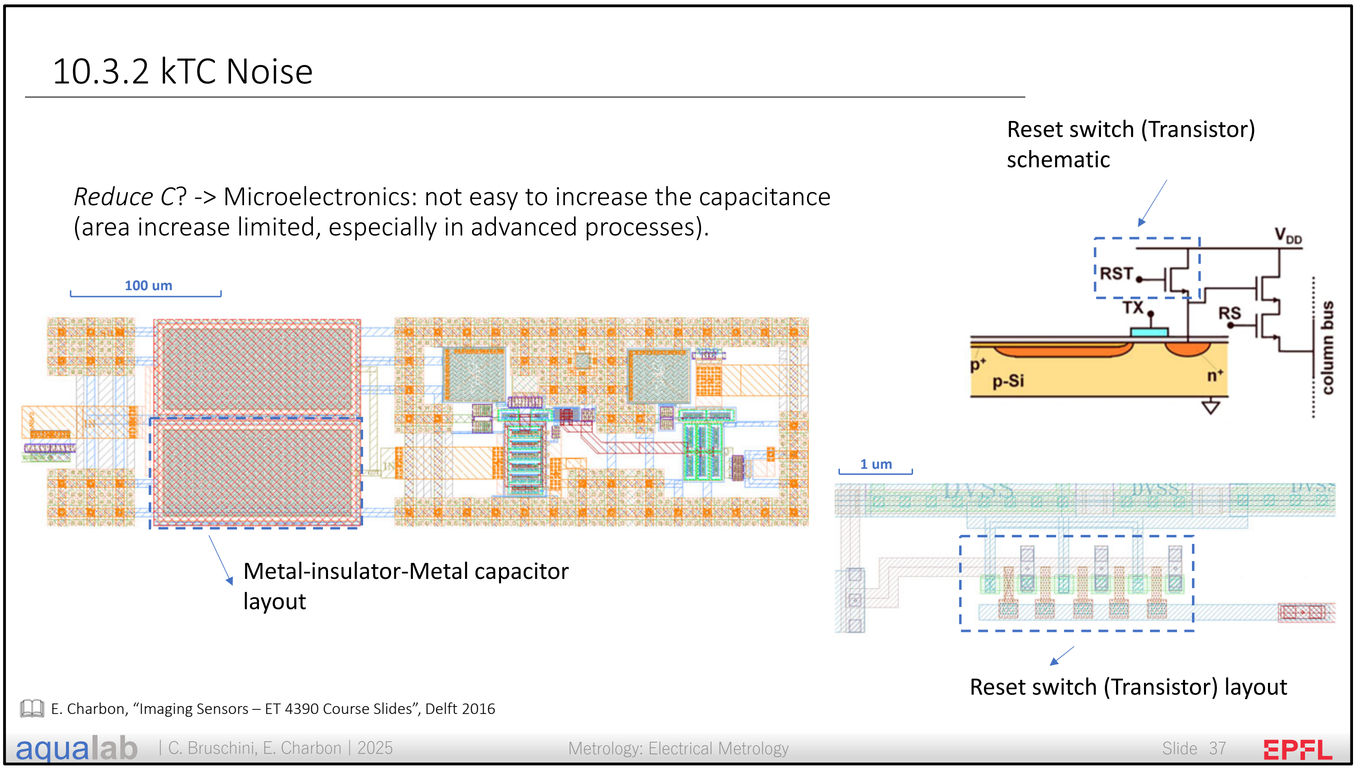
E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

Still on the interpretation of kTC noise:
"Ideal capacitors and inductors do not generate any thermal noise. However, they *do* accumulate noise generated by other sources." - E. Paperno, BGU (IL), *Measurement Theory Fundamentals*, Ch. 5, 2006

Bottom image: an explanation in terms of an equivalent electrical circuit is given by Razavi pp. 228-229. The point is that the voltage source is not a "real" one but depends on the resistor value R (thermal noise source).

Why does the final result not depend on *R*?

- "Intuitively, this is because for larger values of R, the associated noise per unit bandwidth increases while the overall bandwidth of the circuit decreases."
- In other terms, when changing *R* the pole of the RC circuit moves as well, counterbalancing the increase or decrease in PSD.



Right: the reset switch in the pixel of a CMOS imager (top right) is a source of kTC noise. When nothing specific is implemented, such as Correlated Double Sampling (CDS) techniques, this reset noise can become dominant in cameras – see also the two Optical Metrology lectures.

Left: Microelectronics: it is not easy to increase the capacitance on-chip (area increase limited, especially in advanced processes).

10.3.3 Shot (or Poisson) Noise

- Shot noise (derived by Schottky, 1918) results from the fact that the current is *not a continuous flow* but the *sum of discrete pulses*, each corresponding to the transfer of an electron through the conductor.
- Also in photon counting devices (associated to the particle nature of light)!
- May be dominant when few particles (charge carriers, photons) are involved -> large statistical fluctuations *Q*

Example (electronics):

- Generated with current flow through a potential barrier, e.g. in p-n junctions
- *Random* sequence of many *independent* pulses, i.e. *<< shots >>* due to single electrons that swiftly cross the junction depletion layer
  - Random fluctuations of current since *generation* and *recombination* is random
  - Happens in *semiconductor devices* and *vacuum tubes*

S. Cova, "Sensors Signals and Noise, Course Slides", Politecnico Milano 2016; M. de Jong, https://www.lorentz.leidenuniv.nl/beenakker/beenakkr/mesoscopics/topics/noise/noise.html

Shot noise: why do we say "*May be dominant* when few particles are involved"? Can you think of an explanation?

-> Poisson statistics: a low number of events leads to very high variability (in relative terms). Indeed: $standard\ deviation/mean = \sigma/\mu = \sqrt{n}/n = 1/\sqrt{n}$, which increases with decreasing n (number of particles in a given amount of time).

In the case of charge carriers, think for example of electrons "falling down" the potential barrier and crossing the depletion layer in a reversely biased p-n junction. In the case of light, think of a photon flux at low illumination levels.

10.3.3 Shot Noise

- Assume pulses of rate p, charge q, and very short duration T_h (shorter than transition times in circuits – ideally Dirac pulses): the *shot current* has mean value

Ex

$$I = {}^{dQ}/_{dt} = p \cdot q$$

- Shot current has fast fluctuations around the mean, called shot noise (or Schottky noise)

- It can be shown that the *Shot noise* has constant spectral density (bilateral formulation):

$$S_j(\omega) \cong qI \qquad \text{for } \omega \ll 2\pi 10\ GHz$$
$$K_{nn}(\tau) \cong qI\delta(\tau) \qquad \text{for } \tau \gg 100\ ps$$

Q

-> WSS white noise, but in contrast to thermal noise, shot noise *cannot* be reduced by lowering the *temperature*!

S. Cova, "Sensors Signals and Noise, Course Slides", Politecnico Milano 2016

First bullet: think of pulses with rate *p* and charge *q*. [Cova *Noise 2*]

Last bullet:

- White noise up to a certain cut-off frequency, which is for example related to the time taken for an electron to travel through the conductor or junction.

- In terms of autocorrelation function: a perfectly flat spectrum would have a delta function as autocorrelation.

- In this case the autocorrelation function is "delta-like" of width ~100 ps but finite height (the integral should be 1). [Cova *Noise 3*, slides 15-16]
- This becomes the autocorrelation of the Lorentzian spectrum once the current pulses are RC-filtered. [Cova *Noise 3* slide 9]

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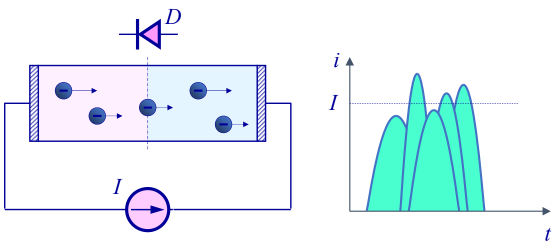
10.3.3 Shot Current

- Example: Shot Noise in a Diode



E. Paperno, BGU (IL), *Measurement Theory Fundamentals*, Ch. 5, 2006 -> K. B. Klaassen, *Electronic measurement and instrumentation*, Cambridge University Press, 1996.

10.3.3 Shot Current in p-n Junction

- In a reverse-biased p-n junction, the shot current is a random sequence of statistically independent Dirac pulses $f(t)$
- It means that the probability for a pulse to occur is independent of the occurrence of other pulses
  - No history
  - $p(t)dt$ is the probability that a pulse starts in $[t, t + dt]$
  - We consider $p(t) = p$ (independent of t) and p equals to I/q

Q

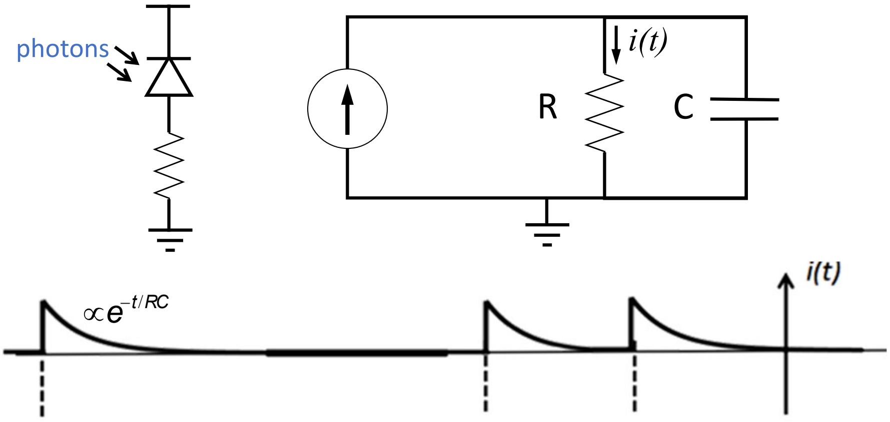
E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

Q: what are these (probability) characteristics reminding you of?

→ **Poisson process**! See also Appendix B, Week 9.

10.3.3 Shot Current in p-n Junction Diode (photocurrent)

- In reality - macroscopically - the white noise is filtered by the parasitics of the junction, thus the Dirac pulses are visible as series of exponential pulses:



E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

We now compare the **microscopic** aspects (focus on individual events) to the **macroscopic** view of the phenomena. See also [*Cova Noise 2* slide 5] and subsequent discussion.

We also transition to the case of a photodiode subject to light (note the term "**photocurrent**" in the slide title) → the Poisson nature of the impinging photons "induces" a photocurrent of the same nature.

10.3.3 Shot Current in p-n Junction Diode (photocurrent)

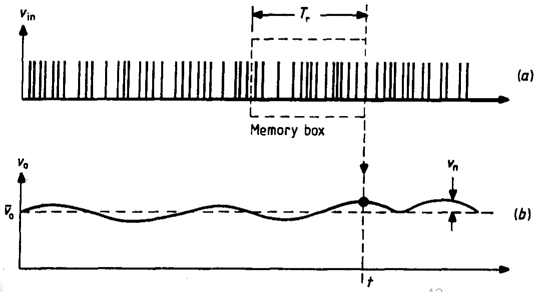
- Photons produce electron-hole pairs with a small probability, known as Quantum Efficiency (QE) *Q*

- QE = percentage of photons impinging on a diode that produce a charge carrier

→ Since the charge of the electron is q = 1.6×10^{-19} Coulomb, and remembering that $C = {}^{Q}/_{V} \rightarrow V = {}^{Q}/_{C}$, the voltage seen across the resistor is at most q/C = 1.6 µV (assuming C = 100 fF and no current goes through R initially)

- Thus, macroscopically it will be seen as continuous noise - the individual fluctuations are not visible anymore, all the more so as the photon flux increases.

Alternative derivation: count the number of quanta within a square time window of width T (equivalent to an integration over time)



M. Van Exter, "Noise and Signal Processing", Univ. Leiden, 2003

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

- First bullet: do you remember what the **Quantum Efficiency** of different photosensors is? In which waveband is silicon sensitive? To what wavelengths do colours correspond?

Microscopic vs. macroscopic:

- "Continuous noise": what happens at the macroscopic level is that we basically go from an idealised current consisting of distinct Dirac pulses at a rate *p*, each one carrying a charge *q*, to a realistic (first order, low-pass filtered -> RC) response which looks continuous at the "usual" time scales. The individual fluctuations are not visible anymore, all the more so as the photon flux increases.

- An alternative, somewhat more general derivation, is based on the counting of the number of quanta within a square time window of width *T* in the image below, equivalent to an integration over time.

____

10.3.3 Shot Noise (photocurrent)

S

- Example: let us compute the photocurrent in a p-n junction for a radiating power of 100nW of red light ($\lambda = 612nm$) onto the photodiode

- The photon flux is:

$$PF_D = \frac{P_D}{hc/\lambda} = \frac{100 \text{ nW}}{3.23 \times 10^{-19}\text{J}} = 3.1 \times 10^{11} \text{ photons/s}$$

$$I = QE \cdot PF_D \cdot q = 35 \text{ nA}$$

$$\text{where } h\text{: Planck's Constant} = 6.626 \times 10^{-34} \text{ J} \cdot s$$

$$\text{Assume } QE = 0.7$$

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

Example of a **photocurrent** calculation. Note how the number of photons can be very large even at quite low power levels!

10.3.3 Shot Current (photocurrent) vs. Poisson Statistics

- The noise is given by the fact that ***photons*** don't reach the diode uniformly, but follow a Poisson statistics: the probability that k photons reach the diode in a given time interval Δt is (λ = rate of photon arrivals = mean number of photons reaching the diode in the time interval Δt):

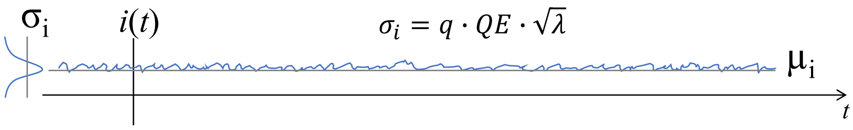
$$P\{X = k\} = \frac{\lambda^k e^{-\lambda}}{k!}$$

- If each carrier is generated by a photon in the photodiode, then the photocurrent also follows Poissonian statistics:

$$i(t) = q \cdot QE \cdot k(t)$$

$$E\{i(t)\} = \mu_i = q \cdot QE \cdot \lambda$$

$$\sigma_i = q \cdot QE \cdot \sqrt{\lambda}$$



- The current will (also) be a random process, i.e. a collection of RVs with Poisson statistics

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016

We emphasize again the case of a photodiode subject to light (note the term "photocurrent" in the slide title) → the Poisson nature of the impinging photons "induces" a photocurrent of the same nature.

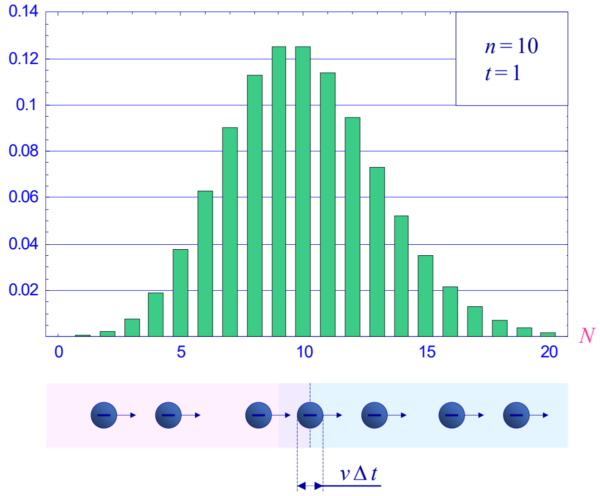
NB: λ is here the rate of photon arrivals, not a wavelength!

We can identify two extreme cases:
- Lower photon flux -> "pure" Poisson distribution.

- Higher photon flux: we would basically see a Gaussian distribution centred around the mean photocurrent value μ_i, with standard deviation σ_i, as indicated by the small image on the left.

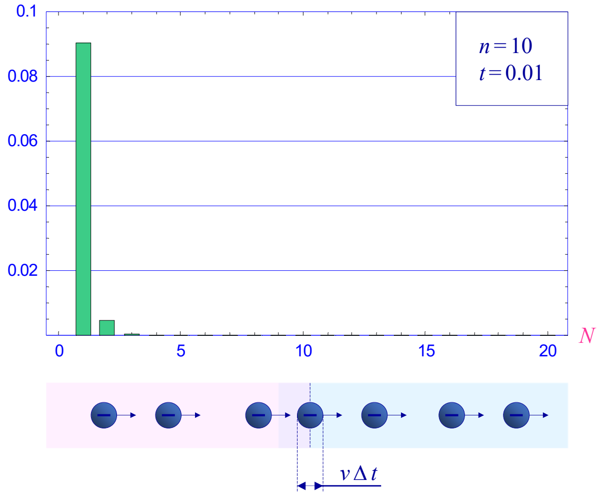
10.3.3 Shot Noise: Poisson Probability Distribution

$$P_n(t) = \frac{(nt)^n}{n!}e^{-nt}$$



10.3.3 Shot Noise: Poisson Probability Distribution

$$P_n(t) = \frac{(nt)^n}{n!} e^{-nt}$$



This plot is obtained using the same average event rate as on the previous slide (*n = 10*) but over a much shorter integration time. The resulting distribution is obviously quite different!

10.3.3 Shot Noise - Summary

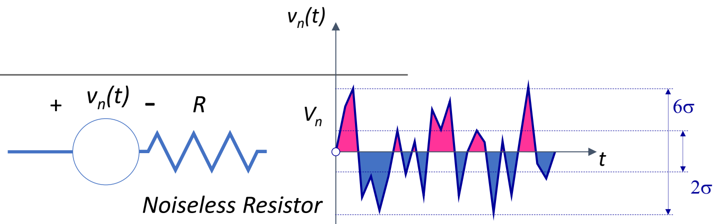
- Through a *p-n* junction (or any other potential barrier), the electrons are transmitted *randomly* and *independently* of each other. Thus the transfer of electrons can be described by Poisson statistics.
- Shot noise is absent in a *macroscopic*, metallic resistor because the ubiquitous inelastic electron-phonon scattering smoothes out current fluctuations that result from the discreteness of the electrons, leaving only thermal noise.
- Shot noise *does* exist in *mesoscopic* (nm) resistors, although at lower levels than in a diode junction. For these devices the length of the conductor is short enough for the electron to become correlated, a result of the Pauli exclusion principle.
  - In this case the electrons are no longer transmitted randomly, but according to *sub-Poissonian* statistics.

M. de Jong, https://www.lorentz.leidenuniv.nl/beenakker/beenakkr/mesoscopics/topics/noise/noise.html (published in ***Physics World, August 1996, page 22***)

Additional sources:

- https://en.wikipedia.org/wiki/Mesoscopic_physics

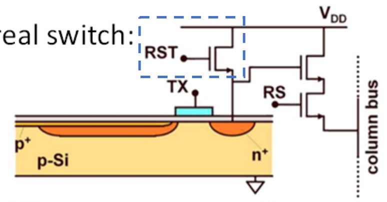
Take-home Messages/W4-2



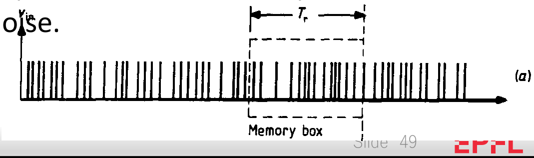
- *Noise Sources:*
  - Thermal noise, e.g. in resistors.
    - White noise. Bilateral PSD: $2kTR$.

Reset switch (Transistor) schematic

  - kTC noise (thermal noise on capacitors), e.g. when opening a real switch:
    - $\overline{V_0^2} = \frac{kT}{C}$



  - Shot (or Poisson noise), e.g. current flow through a potential barrier, or in photon-counting devices. Bilateral PSD: S=qI, WSS white noise.



  - NB: thermal and shot noise are irreducible.

Second recap section: we summarise here the main definitions, results and examples discussed so far.

10.3.4 Flicker (1/f) Noise

- Thermal noise and shot noise are irreducible (ever present) forms of noise. They define the minimum noise level or the 'noise floor'. Many devices generate additional or *excess noise*.
- The most general type of excess noise is $1/f$ or flicker noise. This noise has approximately $1/f$ power spectrum (equal power per decade of frequency) and is sometimes also called pink noise. *Ex*
- $1/f$ noise is usually related to the fluctuations of the device properties caused, for example, by electric current in resistors and semiconductor devices.
- Curiously enough, $1/f$ noise is present in nature in unexpected places,
  - the speed of ocean currents
  - the flow of traffic on an expressway
  - the loudness of a piece of classical music versus time,
  - and the flow of sand in an hourglass.
- *No unifying principle* has been found for all the $1/f$ noise sources.

E. Paperno, BGU (IL), *Measurement Theory Fundamentals*, Ch. 5, 2006 -> P. Horowitz and W. Hill, *The art of electronics*, Second Edition, Cambridge University Press, 1989.

The origin of **1/f noise** is still the subject of debates. Some additional sources:

- [*Cova 1/f NOISE and HPF 1*]
- "Curiously enough…": MDPI Appl. Sci. **2018**, 8, 1685; doi:10.3390/app8091685
- https://electronics.stackexchange.com/questions/533236/what-causes-flicker-or-1-f-noise refers to *Horowitz, Hill, The Art of Electronics* (probably), 8.1.3, quoting:

"Other noise-generating mechanisms often produce 1/f noise, examples being base-current noise in transistors and cathode current noise in vacuum tubes. Curiously enough, 1/f noise is present in nature in unexpected places, e.g., the speed of ocean currents, the flow of sand in an hourglass, the flow of traffic on Japanese expressways, and the yearly flow of the Nile measured over the last 2,000 years. If you plot the loudness of a piece of classical music versus time, you get a 1/f spectrum! No unifying principle has been found for all the 1/f noise that seems to be swirling around us, although particular sources can often be identified in each instance."

- Note on *Milotti's 2002* opinion on 1/f noise, in "*1/f* noise: a pedagogical review":
"In this review we have studied several mechanisms that produce fluctuations with a *1/f* spectral density: do we have by now an "explanation" of the apparent

universality of flicker noises? Do we understand *1/f* noise? My impression is that there is no real mystery behind *1/f* noise, that there is no real universality and that in most cases the observed *1/f* noises have been explained by beautiful and mostly *ad hoc* models."

10.3.4 Flicker Noise

- In electrical and electronic devices, flicker noise occurs only when electric current is flowing.
- In semiconductors, flicker noise usually arises due to traps, where the carriers that would normally constitute dc current flow are held for some time and then released.
- Although bipolar, JFET, and MOSFET transistors have flicker noise, it is a significant noise source in MOS transistors, whereas it can often be ignored in bipolar transistors.
- Summarising, it is observed in all electronic devices:
  - Strong in MOSFETs
  - Moderate in Bipolar transistors BJTs
  - Moderate in carbon resistors
  - Ultra-weak in metal-film resistors

E. Paperno, BGU (IL), *Measurement Theory Fundamentals*, Ch. 5, 2006 -> R. B. Northrop, *Introduction to instrumentation and measurements*, second edition, CRC Press,2005 & D. A. Jones and K. Martin, *Analog integrated circuit design*, John Wiley & Sons, 1997.

E. Charbon, “Imaging Sensors – ET 4390 Course Slides”, Delft 2016; S. Cova, “*Sensors Signals and Noise – Course Slides*”, Politecnico di Milano 2016

1/f Noise in resistors:

“For carbon composition resistors, this is typically 0.1 µV to 3 µV of rms noise per volt applied across the resistor. Metal film and wirewound resistors have about 10 times less noise” (https://www.thinksrs.com/downloads/pdfs/applicationnotes/AboutLIAs.pdf)

10.3.4 Flicker Noise

- $1/f$ noise arises from physical processes that generate a random superposition of elementary pulses with random pulse duration ranging from **very short to very long**
- Q E.g. in MOSFETs $1/f$ noise is due to:
  - Carriers traveling in the conduction channel are captured by local trap
  - trapped carriers are later released by the level with a random delay
  - the level lifetime (=mean delay) strongly depends on how distant from the silicon surface (= from the conduction channel) is the level in the oxide
  - trap levels are distributed from very near to very far from silicon, lifetimes are correspondingly distributed from very short to very long

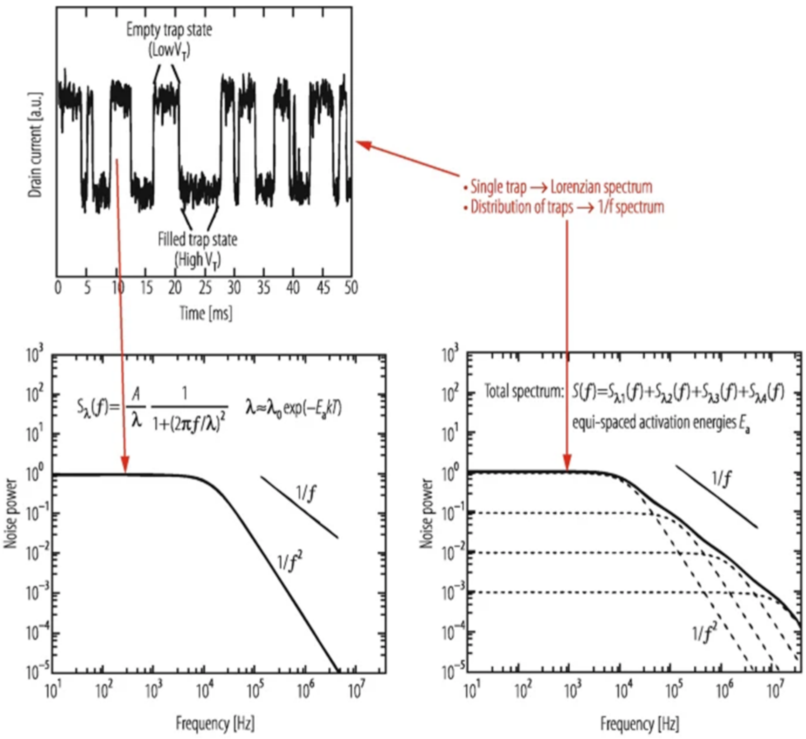
Example: *distributed trapping model* (superposition of uniform exponential relaxation processes = superposition of Lorentzian PSDs, corresponding to RTS behaviours, with uniform distribution of lifetimes)…

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016; S. Cova, "*Sensors Signals and Noise – Course Slides*", Politecnico di Milano 2016

→ https://link.springer.com/chapter/10.1007/978-3-030-35318-6_10 :

10.3.3 Random Telegraph Noise and 1/*f* Noise

A noise spectrum very close to $1/|f|$ can be generated by superposition of relaxation processes with uniform distribution of life times, as illustrated in Fig. 10.11. The relaxation process is described by $U(t)\exp(-t/\tau)$, which represents a step change with exponential decay. Trapping-detrapping in semiconductors is one such possible mechanism for generation of $1/|f|$ noise. Since a simple RC integrator has the same response, a hardware filter which transforms white noise into $1/|f|$ noise can be made requiring about one time constant (one RC circuit) per decade of frequency, as shown in Ref. [16].



10.3.4 Flicker Noise

- Basic distinction between $1/f$ and white noise: time span of interdependence between samples
  - for white noise: samples are uncorrelated even at short time distance
  - for $1/f$ noise: samples are strongly correlated even at long time distance
- Often the relation is more complex:

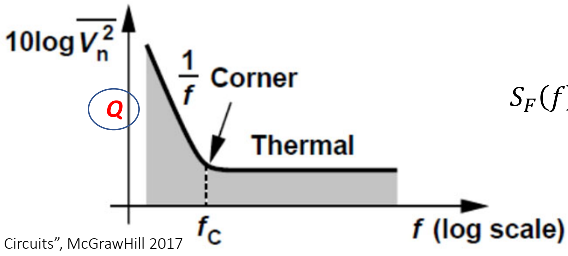
$$S_F(\omega) \propto \frac{1}{|f|^{\alpha}}, 0.8 < \alpha < 1.2$$

E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016; S. Cova, "*Sensors Signals and Noise – Course Slides*", Politecnico di Milano 2016

See also: [*Cova 1/f NOISE and HPF 1*]

10.3.4 Flicker Noise

- At low frequencies, the flicker noise power approaches infinity
- At very slow rates, flicker noise becomes indistinguishable from thermal drift or aging of devices
- Noise component below the lowest frequency in the signal of interest does not corrupt it significantly
- Intersection point of thermal noise and flicker noise spectral densities is called "corner frequency" f_C, usually between 10 Hz and 1 MHz



$$S_F(f) = \frac{P}{|f|}, \quad \overline{n_F^2} = \int_0^\infty \frac{P}{|f|} df$$

(spectral density, unilateral; noise power)

📖 B. Razavi, "Design of Analog CMOS Integrated Circuits", McGrawHill 2017

📖 E. Charbon, "Imaging Sensors – ET 4390 Course Slides", Delft 2016; S. Cova, "*Sensors Signals and Noise – Course Slides*", Politecnico di Milano 2016

- Source of first 3 bullets: [Razavi 7.2.2]
- In the equations: $S_F(f)$ is the Spectral density (single-sided), P the noise power.

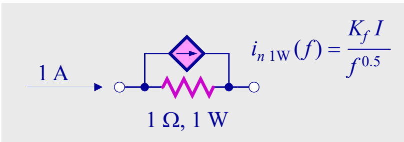
Q: if we have a very low frequency value (e.g. sub-Hz), what is the problem in doing such a measurement?

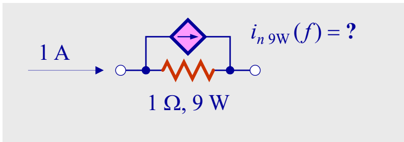
→ Long integration times! [Razavi: *A noise component at 0.01 Hz varies significantly in roughly 10 s (one-tenth of the period) and one at 10−6 Hz in roughly one day.*]

→ Plus still problematic to distinguish the true signal fluctuations from thermal drift or aging. For example, it is not always easy to keep the temperature of the circuit or device stable, e.g. because it can heat up during the measurement.

10.3.4 Flicker Noise

- [Resistors] Flicker noise is directly proportional to the dc (or average) current flowing through the device (K_f = constant depending on type of material)
- The spectral power density of $1/f$ noise in resistors is in inverse proportion to their power dissipating rating.
- This is so, because the resistor current density decreases with the square root of its power dissipating rating.
- Exercise: compare $1/f$ noise in 1 Ω, 1 W and 1 Ω, 9 W resistors for the same 1 A dc current…





E. Paperno, BGU (IL), *Measurement Theory Fundamentals*, Ch. 5, 2006 -> R. B. Northrop, *Introduction to instrumentation and measurements*, second edition, CRC Press,2005 & D. A. Jones and K. Martin, *Analog integrated circuit design*, John Wiley & Sons, 1997.

-> P is here actually the power dissipation rating (not the power spectral density!), which is a technological parameter corresponding to the max power which the resistor can dissipate, not the actual power.

Two resistors with different power dissipation ratings are built differently.

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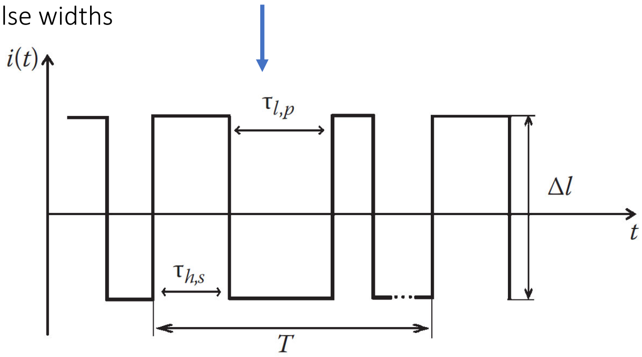
10.3.5 RTS Noise

- Burst noise is another type of noise at low frequencies.
- Recently, this noise was described as RTS (Random Telegraph Signal) noise.
- With given biasing condition of a device the magnitude of pulses is constant, but the switching time is random.
- The burst noise looks, on an oscilloscope, like a square wave with the constant magnitude, but with random pulse widths



A. Konczakowska & B. M. Wilamowski, *Noise in Semiconductor Devices*, in *Fundamentals of Industrial Electronics*, Ch. 11, 2011

Additional sources:

https://www.sciencedirect.com/topics/engineering/signal-output -> Infrared Thermography Basics *Carosena Meola, ... Giovanni Maria Carlomagno, in Infrared Thermography in the Evaluation of Aerospace Composite Materials, 2017,* 3.4.4 Detector Performance

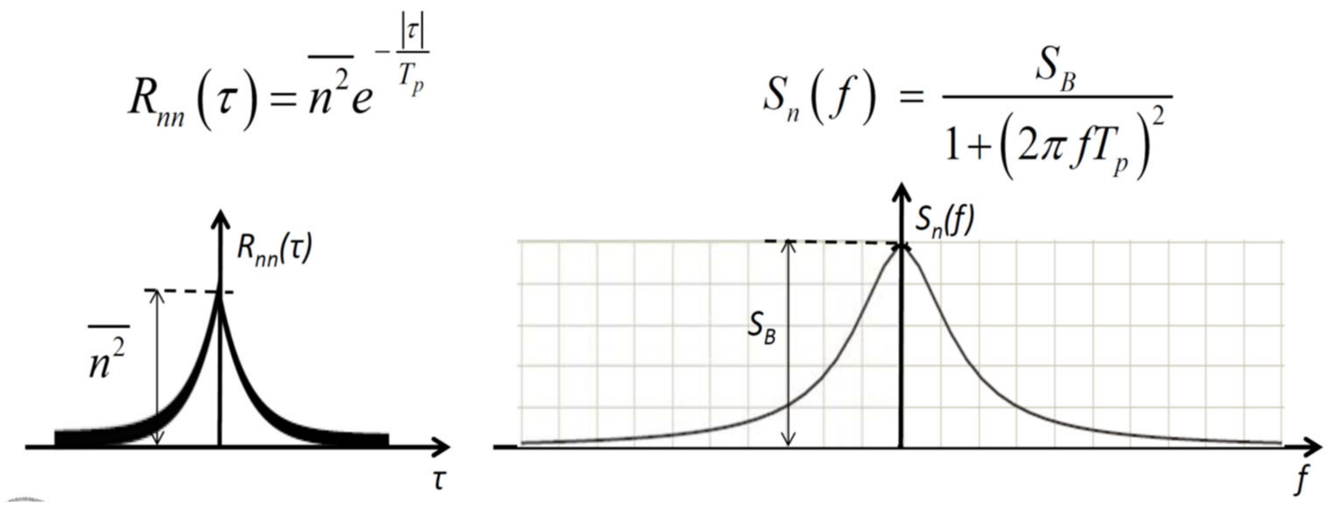
10.3.5 RTS Noise

- This kind of noise is due to generation/recombination effects and trapping in semiconductors.
- Telegraph/Popcorn Noise is bias and frequency dependent
- *Lorentzian spectrum*: The Noise spectral density function of the RTS noise has a similar form like generation-recombination noise (f_{RTS} = RTS noise corner frequency, below this frequency spectrum the RTS noise is flat)

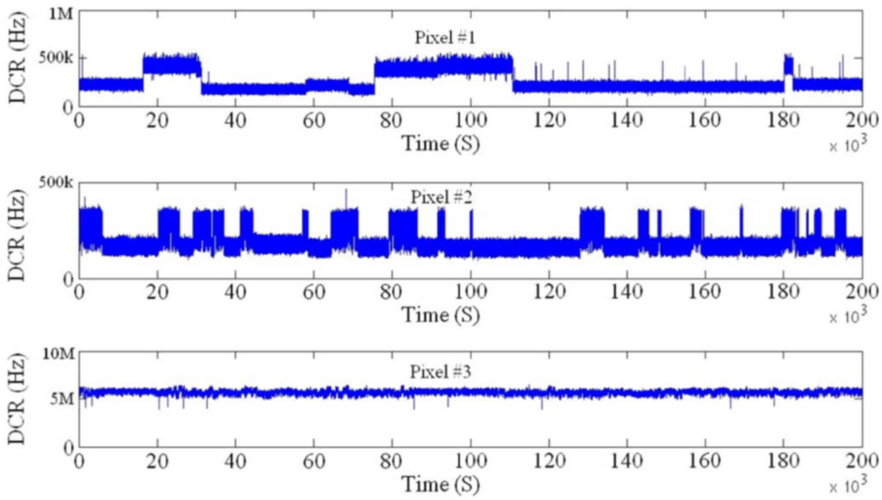
$$S_{RTS}(f) = C\frac{4(\Delta I)^2}{1+\left(2\pi f/f_{RTS}\right)^2}$$

📖 A. Konczakowska & B. M. Wilamowski, *Noise in Semiconductor Devices*, in *Fundamentals of Industrial Electronics*, Ch. 11, 2011

Below: example of a Lorentzian spectrum and its autocorrelation function. [*Cova Noise 3* slide 9]



10.3.5 RTS Noise – Example, non-stationary



Q

(radiation tolerance characterization, aging emulation, dose-rate effects)

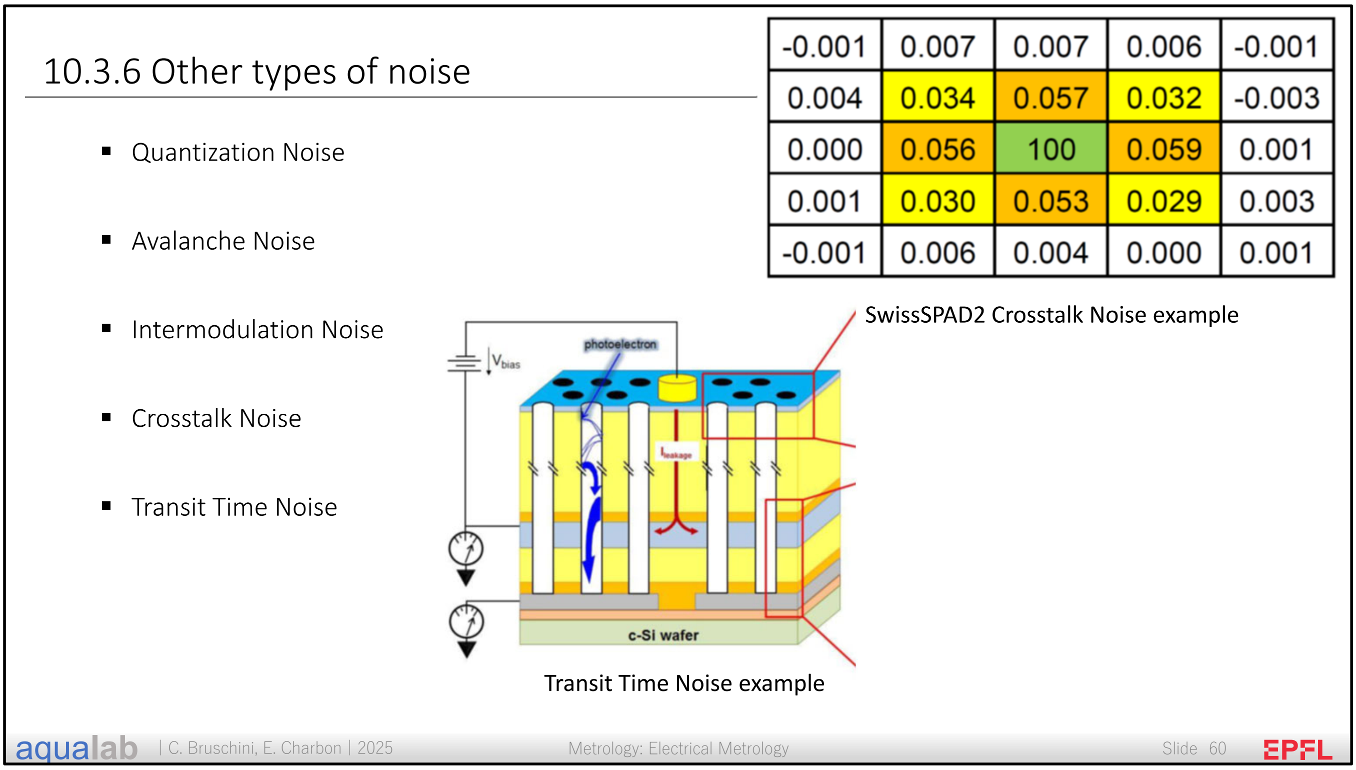
M. A. Karami et al., *Random Telegraph Signal in Single-Photon Avalanche Diodes*, International Image Sensor Workshop, Bergen, 2009

An example of a non-stationary Random Process is provided by the noise behaviour of irradiated SPADs, which can exhibit RTS behaviour over long periods of time. The noise level, or Dark Count Rate (DCR), does basically jump between two or more different levels. This is shown here for 3 different devices of the same type.

10.3.5 RTS Noise

- This noise has significant effect at low frequencies.
- It is a function of temperature, induced mechanical stress, and also radiation.
- In audio amplifiers, the burst noise sounds as random shots, which are similar to the sound associated with making popcorn.
- Obviously, BJTs with large burst noise must not be used in audio amplifiers and in other analog circuitry.
- It is now assumed that devices fabricated with well-developed and established technologies do not generate RTS noise.
- This is unfortunately not true for modern nanotransistors and devices fabricated with other than silicon materials.

A. Konczakowska & B. M. Wilamowski, *Noise in Semiconductor Devices*, in *Fundamentals of Industrial Electronics*, Ch. 11, 2011



Examples of other noise sources:

- **Quantization noise** -> next lecture (ADCs, TDCs).

- **Cross-talk:** think of the situation between pixels of an imagers, e.g. SwissSPAD2, as shown in the crosstalk probability plot in the top right image.

Q: why are there negative values!?!

→ Think of the statistical count rate variability, and the need to subtract the background between measurements, due to the DCR and ambient background.

- **TTS** (Transit Time Spread): think of the initial example from a multichannel plates detector (slides 12-13). This is basically linked to the variability in transit time from one event to the other.

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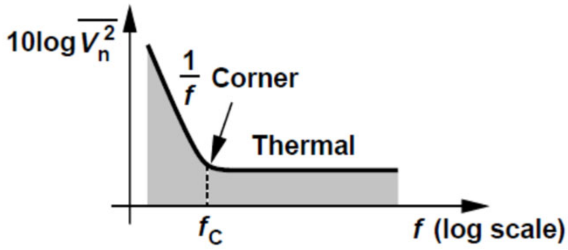
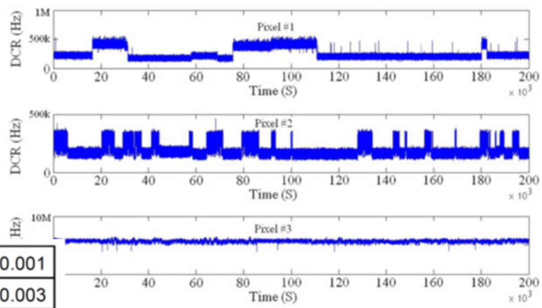
Take-home Messages/W4-3

- *Noise Sources:*
  - Flicker (1/f) noise
    - $S_F(\omega) \propto \frac{1}{|f|^{\alpha}}, 0.8 < \alpha < 1.2$
  - RTS noise:
    - $S_{RTS}(f) = C \frac{4(\Delta I)^2}{1+\left(\frac{2\pi f}{f_{RTS}}\right)^2}$
  - Other noise sources





-0.001	0.007	0.007	0.006	-0.001
0.004	0.034	0.057	0.032	-0.003
0.000	0.056	100	0.059	0.001
0.001	0.030	0.053	0.029	0.003
-0.001	0.006	0.004	0.000	0.001

Final recap section: we summarise here the main definitions, results and examples discussed so far.

Acknowledgments

- Bedirhan Ilik (TA 2019)
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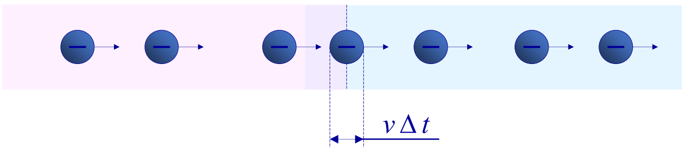
Appendix A - 10.3.3 Shot Noise vs Poisson Statistics

[Derivation of Poissonian statistics]

- Start from defining n as the average number of electrons crossing the p-n junction of a diode during one second, hence, the average electron current $I = qn$.
- Assume that the probability of two or more electrons crossing simultaneously is negligibly small, $P_{>1}(dt) = 0$.
- This allows us to define the probability that an electron crosses the junction in the time interval

$$dt = (t, t + dt) \quad \text{as} \quad P_1(dt) = n\,dt$$

(dt is approaching the time taken for an electron to travel across the junction, < 1 ns).



E. Paperno, BGU (IL), *Measurement Theory Fundamentals*, Ch. 5, 2006

10.3.3 Shot Noise vs Poisson Statistics

Next, we derive the probability that no electrons crosses the junction in the time interval $(0, t + d\,t)$:

$$P_0(t + d\,t) = P_0(t)\,P_0(d\,t) = P_0(t)\,[1 - P_1(d\,t)] = P_0(t) - P_0(t)\,n\,d\,t.$$

This yields:

$$\frac{dP_0}{dt} = -nP_0$$

with the obvious initiate state $P_0(0) = 1$.

E. Paperno, BGU (IL), *Measurement Theory Fundamentals*, Ch. 5, 2006

10.3.3 Shot Noise vs Poisson Statistics

The probability that an electron crosses the junction in the time interval $(0, t + \mathrm{d}\, t)$

$$P_1(t + d\,t) = P_1(t)\, P_0(d\,t) + P_0(t)\, P_1(d\,t)$$
$$= P_1(t)\,(1 - n\,d\,t) + P_0(t)\, n\,d\,t\,.$$

This yields

$$\frac{dP_1}{dt} = -nP_1 + nP_0$$

with the obvious initiate state $P_1(0) = 0$.

E. Paperno, BGU (IL), *Measurement Theory Fundamentals*, Ch. 5, 2006

10.3.3 Shot Noise vs Poisson Statistics

- In the same way, one can obtain the probability of *N* electrons crossing the junction:

$$\begin{cases} \dfrac{dP_n}{dt} = -nP_n + nP_{n-1} \\ P_n(0) = 0 \end{cases}$$

By substitution, one can verify that

$$P_n(t) = \frac{(nt)^n}{n!} e^{-nt},$$

which corresponds to the Poisson probability distribution.

E. Paperno, BGU (IL), *Measurement Theory Fundamentals*, Ch. 5, 2006